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General description of Dirac spin-rotation effect with relativistic factor

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The Mashhoon rotation-spin coupling is studied by means of the parallelism description of general relativity. The relativistic rotational tetrad is exploited, which results in the Minkowski metric, and the torsion axial-vector and Dirac spin coupling will give the Mashhoon rotation-spin term. For the high speed rotating cases, the tangent velocity constructed by the angular velocity Ω multiplying the distance r may exceed over the speed of light c , i.e., $\Omega r \geq c$, which will make the relativistic factor γ infinity or imaginary. In order to avoid this “meaningless” difficulty occurred in γ factor, we choose to make the rotation nonuniform and position-dependent in a particular way, and then we find that the new rotation-spin coupling energy expression is consistent with the previous results in the low speed limit.

Keywords: torsion, parallelism, rotation-spin(1/2), noninertial effect

1. Introduction

On the rotation-spin coupling, the phenomenon of rotation-spin coupling illustrates the inertia of intrinsic spin, and its existence was first proposed by Mashhoon^{1,2,3,4,5}. An experiment to test for the existence of this term has been studied by Mashhoon *et al.* and many others^{6,7,8,9,10}. It then follows easily¹¹ that the coupling of spin with rotation holds also at the relativistic level. It should be noted, however, that this only holds in *Minkowski spacetime*¹².

The straightforward theoretical derivations of the inertial effect of Dirac particle have been performed by Hehl and his collaborators^{13,14}, and these treatments have been extended in several directions by a number of investigators^{15,16}, however, where the relativistic factor has not been considered. Hehl and Ni¹³ have calculated spin coupling Hamiltonian in an arbitrary noninertial frame, one subject both to rotation and to acceleration, and rotation-spin coupling is a result of their special case of zero acceleration. While, with regard to the existed references, we stress that the metric tensor implied from the exploited tetrad by many authors^{12,13,15,17,18,19} is not a Minkowski one, or the spacetime curvature exists, which seems to declaim that the rotation produces the spacetime curvature. Moreover, there is no role of relativistic factor in their tetrads, and they have to constrain their discussions within the light cylinder ($R_L = c/\Omega$)¹² because of the velocity limit of speed of light. In principle, the description of rotation-spin might be in the

special relativistic level and application of this effect could be extended anywhere in the rotational frame, within or beyond the light cylinder.

On the torsion-spin coupling, it is one of most important characteristics for the torsion gravity to compare with the general relativity (GR). As a special version of torsion gravity, the parallelism of general relativity (PGR) has been pursued by a number of authors^{21,22,23,24,25,26}, where the spacetime is characterized by the torsion tensor and the vanishing curvature, named Weitzenböck spacetime²¹, which can be reduced from the Riemann-Cartan spacetime where the generalized metric-affine theory of gravitation is constructed^{27,28,29,30}. PGR will be equivalent to GR with the convenient choice of the parameters of the Lagrangian²¹, however based on the tetrad PGR is a powerful and a natural tool to describe the Dirac field³¹.

In notation of the symbols, we will use the Greek alphabet ($\mu, \nu, \rho, \dots = 1, 2, 3, 4$) to denote tensor indices, that is, indices related to spacetime. The Latin alphabet ($a, b, c, \dots = 1, 2, 3, 4$) will be used to denote local Lorentz (or tangent space) indices. The tensor and local Lorentz indices can be changed into each other with the use of the tetrad e^a_μ , which satisfy

$$e^a_\mu e_a^\nu = \delta_\mu^\nu \quad ; \quad e^a_\mu e_b^\mu = \delta^a_b . \quad (1)$$

A nontrivial tetrad field can be used to define the linear Cartan connection^{21,23}

$$\Gamma^\sigma_{\mu\nu} = e_a^\sigma \partial_\nu e^a_\mu , \quad (2)$$

with respect to which the Cartan connection is^{21,23}

$$T^\sigma_{\mu\nu} = \Gamma^\sigma_{\mu\nu} - \Gamma^\sigma_{\nu\mu} . \quad (3)$$

The metric is

$$g_{\mu\nu} = \eta_{ab} e^a_\mu e^b_\nu , \quad (4)$$

where η^{ab} is the metric in flat space with the line element

$$d\tau^2 = g_{\mu\nu} dx^\mu dx^\nu , \quad (5)$$

The irreducible torsion axial-vector can then be constructed as^{21,23,32}

$$A_\mu = \frac{1}{6} \epsilon_{\mu\nu\rho\sigma} T^{\nu\rho\sigma} , \quad (6)$$

with $\epsilon_{\mu\nu\rho\sigma}$ being the completely antisymmetric tensor normalized as $\epsilon_{0123} = \sqrt{-g}$ and $\epsilon^{0123} = \frac{1}{\sqrt{-g}}$, where g is the determinant of metric.

The spacetime dynamic effects on the spin is incorporated into Dirac equation through the “spin connection” appearing in the Dirac equation in gravitation^{21,32}. In Weitzenböck spacetime, as well as the general version of torsion gravity, it has been shown by many authors^{21,22,33,34,35,36,37,38,39,40} that the spin precession

of a Dirac particle is intimately related to the torsion axial-vector, and it is interesting to note that the torsion axial-vector represents the deviation of the axial symmetry from the spherical symmetry²².

$$\frac{d\mathbf{S}}{dt} = -\frac{3}{2}\mathbf{A} \times \mathbf{S}, \quad (7)$$

where \mathbf{S} is the semiclassical spin vector of a Dirac particle, and \mathbf{A} is the spacelike part of the torsion axial-vector. Therefore, the corresponding extra Hamiltonian energy is of the form,

$$\delta H = -\frac{3}{2}\mathbf{A} \cdot \mathbf{S}. \quad (8)$$

Based on the torsion-spin coupling above, the Lense-Thirring precession of Dirac spin has been obtained³² by the standard Kerr tetrad, and the rotation-spin precession can be achieved as well^{16,20}.

The purpose of the paper is constructing the connection between the torsion-spin coupling and the rotation-spin coupling, and especially we want to cope with the realistic physical conditions that includes the position outside the light cylinder ($R_L = c/\Omega$, see also¹²) or the case of the tangent velocity of the disk exceeding over the speed of light c . For this end, the rotational tetrad is defined and the resultant metric is a Minkowski one, then the rotation-spin will be derived from the torsion-spin inferred from the systematical Dirac equation treatment. The motivation of the paper has two aspects, we firstly demonstrate that the rotation-spin can be described by the special torsion gravity, the parallelism description of general relativity, which possesses the deep theoretical foundations, and secondly the correct or effective description of rotation-spin will help us recognize the new applications of torsion conception. On the latter, we may be confident of that we are extending the geometric-dynamics from the gravitation (Riemannian curved spacetime) to the rotation (Cartan torsional spacetime).

The organization of the paper is as follows: in section 2, we discuss the rotation-spin coupling by introducing the position-dependent angular velocity, and the new rotation-spin coupling formula is derived. The results and discussions are presented in the last section.

2. The derivation of rotation-spin effect

Now we discuss the Dirac equation in the rotational coordinate system with the polar coordinates (t, r, ϕ, z) with the rotating angular velocity Ω set in z -direction. The tetrad can be expressed by the dual basis of the differential one-form^{13,42} through choosing a coframe of the rotational coordinate system²⁰,

$$d\vartheta^0 = \gamma[cdt - (\Omega r/c)(rd\phi)], \quad (9)$$

$$d\vartheta^1 = dr, \quad (10)$$

$$d\vartheta^2 = \gamma[(rd\phi) - (\Omega r/c)cdt], \quad (11)$$

$$d\vartheta^3 = dz, \quad (12)$$

where the relativistic factor $\gamma(r) = 1/\sqrt{1 - (\Omega r/c)^2}$. If Ωr is much less than the speed of light c , then we have $\gamma = 1$ and the classical coframe expression is recovered, which is same as those applied in the existed references^{13,16}. Therefore Eq.(9) and Eq.(11) is a generalized relativistic coframe expression for any rotation velocity, low or high energy cases. However, if $\Omega r/c > 1$, then the relativistic factor will be meaningless, which will happen in the case of either Ω or r being too large. **Therefore, in order to overcome this difficulty, we make choice of a nonuniform and position-dependent angular velocity, written as**

$$\Omega(r) = \frac{\Omega}{1 + (\Omega r/c)}. \quad (13)$$

Thus, the line velocity at radius r $v(r) = \Omega(r)r$ will not exceed over the speed of light. If $r \rightarrow \infty$ or $\Omega r \rightarrow \infty$, we have $v(r) \rightarrow c$. $\Omega(r) < \Omega$ is implied from Eq.(13) and means that the angular velocity at the center of rotating object is higher than that of far apart. While, Eq.(13) help us overcome the infinity difficulty in describing the line velocity of rotating system by considering the limited propagation velocity in the non-rigid disk. The relativistic factor is now modified as $\gamma(r) = 1/\sqrt{1 - (v(r)/c)^2}$.

The tetrad can be obtained with the subscript μ denoting the column index (c.f. ^{13,16}) through replacing the original constant Ω of $\Omega(r)$, with the conventional usage of the unit of speed of light $c=1$,

$$e^a{}_\mu = \begin{pmatrix} \gamma & 0 & -\gamma\Omega(r)r^2 & 0 \\ 0 & 1 & 0 & 0 \\ -\gamma\Omega(r)r & 0 & \gamma r & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (14)$$

with its inverse

$$e_a{}^\mu = \begin{pmatrix} \gamma & 0 & \gamma\Omega(r) & 0 \\ 0 & 1 & 0 & 0 \\ \gamma\Omega(r)r & 0 & \gamma/r & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (15)$$

We can inspect that the tetrad expressions Eqs.(14) and (15) satisfy the orthogonal conditions in Eqs.(1) and (4). From the tetrad given above, we have the line element and the implied metric,

$$\begin{aligned} d\tau^2 &= \eta_{ab} d\vartheta^a \otimes d\vartheta^b = g_{\mu\nu} dx^\mu dx^\nu \\ &= dt^2 - (dr^2 + r^2 d\phi^2 + dz^2), \end{aligned} \quad (16)$$

and with the determinant of the metric

$$g = \det|g_{\mu\nu}| = -r^2. \quad (17)$$

We find that the metric in Eq.(16) is a Minkowski one⁴¹, resulting in the null Riemannian and Cartan curvatures. Although the curvature vanishes, the torsion

(field) may have nonzero components because the torsion is determined by the tetrad and not by the metric. In other words, the basic element in the parallelism description of general relativity is the tetrad and the metric exploited is just a by-product^{21,23,24,25,27,30}. From Eqs.(14) and (15), we can now construct the Cartan connection, whose components to contribute to the non-vanishing torsion axial-vectors are:

$$\Gamma^2_{01} = -(\gamma^2 f)\Omega/r = -\Omega(r)/r, \quad \Gamma^0_{21} = -(\gamma^2 f)\Omega r = -\Omega(r)r, \quad (18)$$

with the factor definition

$$f = 1 - \Omega(r)r. \quad (19)$$

The corresponding non-vanishing torsion components contributed to the axial torsion-vectors are:

$$T^2_{01} = -\Omega(r)/r, \quad T^0_{21} = -\Omega(r)r, \quad (20)$$

and the non-vanishing torsion axial-vectors are consequently

$$A_3 = \frac{2}{3}\Omega(r), \quad A_k = 0, k = 0, 1, 2. \quad (21)$$

As shown, $A_1 = A_2 = 0$ is on account of the Z-axis symmetry which results in the cancelling of the r and ϕ components, and then generally we can write the torsion axial-vector in the usual vector expression $\mathbf{A} = \frac{2}{3}\mathbf{\Omega}(r)$. From the spacetime geometry view, the torsion axial-vector represents the deviation from the spherical symmetry, which will disappear in the spherical case (Schwarzschild spacetime for instance) and occurs in the axisymmetry case (Kerr spacetime for instance)^{22,32}. If the physical measurement is performed in the rotating frame, the time dt is taken as the proper time through setting the null space difference. Therefore the torsion axial-vector corresponds to an inertia field with respect to Dirac spin, which is now clearly expressed as a precession equation by Eq.(7),

$$\frac{d\mathbf{S}}{dt} = -\mathbf{\Omega}(r) \times \mathbf{S}. \quad (22)$$

The additive energy in the rotating frame is

$$\delta H = -\mathbf{\Omega}(r) \cdot \mathbf{S}, \quad (23)$$

which is a similar form as that expected by Mashhoon (c.f. Ref.^{2,4}) except the introduced varied angular velocity with the radius r for the sake of avoiding the infinite line velocity.

3. Discussions and conclusions

The obtained results of this paper are summarized in the following:

The Mashhoon rotation-spin coupling has been derived from the torsion-spin coupling by means of the parallelism of general relativity, where Dirac equation in the external field is treated by the introduced

rotational tetrad. In the relativistic rotational tetrad, which has resulted in the flat metric, the relativistic factor γ is introduced, but it needs the tangent velocity of the observer to be less than the speed of light. In order to satisfy the need of the upper limit of velocity, we choose to make the rotation nonuniform and position-dependent in a way as described in Eq.(13). Then this choice makes the tangent velocity expression of rotating observer not exceed the speed of light at any positions, within or outside the light cylinder.

On the description of the Mashhoon rotation-spin coupling in our parallelism version, the choice of tetrad will produce a Minkowski metric, a flat spacetime one, which arises a new conclusions on the spacetime torsion and curvature. The rotational field can be an independent source for the production of torsion, and even the gravitational field switches off. The existence of torsion in the flat spacetime, or the Minkowski spacetime, can help us change the point of view that there is no torsion in the flat spacetime ^{21,23,27}.

The extra additive energy of the Mashhoon rotation-spin coupling is expressed to be $\Omega(r)\hbar/2$, so in the low speed limit of the present-day laboratory $\Omega r \ll c$, this coupling energy is still $\Omega\hbar/2$ as obtained by Mashhoon before ^{1,2,3}. However, at the light cylinder of rotating neutron star, $R_L = c/\Omega \sim 10^7(cm)(\Omega/500)^{-1}$, the Mashhoon rotation-spin coupling will be half of that on the surface of star, then we cannot sure if this effect can be detected in the observations of the electromagnetic emissions in the rotating X-ray neutron stars and black holes ⁴³. The quasi periodic oscillations of X-ray flux have been discovered in many X-ray neutron star and black hole systems since the launch of RXTE satellite ⁴³, however these phenomena have not yet been explained. Whether or not some oscillations of the X-ray energy are involved in the Mashhoon rotation-spin coupling still needs a thorough investigation.

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